Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2014
Final Exam
Professor: Victor G. LeBlanc
Time limit: 3 hours. Closed books.

Name:________________________ ID Number:________________

Instructions

- This is a closed book exam. Furthermore, all cell phones, pagers or any other electronic or communication devices are forbidden. **The only calculators which are allowed are Texas Instruments TI-30, TI-34, Casio fx-260 and fx-300, scientific and non programmable.**
- This exam has 15 pages and you have 3 hours to complete it.
- Read each question carefully before answering.
- Questions 1 to 10 are multiple choice questions. These questions are worth 2 points each and no partial marks are possible. **Please write your answers in the corresponding boxes in the grid below entitled "Answers to multiple choice Qs".**
- Questions 11 to 16 are long answer questions and are worth 5 marks each, so organize your time accordingly. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Do not unstaple the test. Good luck!

Answers to multiple choice Qs

1	2	3	4	5	6	7	8	9	10

Grid below is used for grading (do not write in this grid)

MCQ	11	12	13	14	15	16	Total
/20	/5	/5	/5	/5	/5	/5	/50

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1. Consider the function $f(x, y) = x^3y + 2x^2y + x - y$. Which of the following expressions corresponds to the tangent plane to the graph of $z = f(x, y)$ at the point $(x, y, z) = (1, 2, f(1, 2))$?

A. $z = 15x + 2y$

B. $z = (3x^2y + 4xy + 1)(x - 1) + (x^3 + 2x^2 - 1)(y - 2) + 5$

C. $z = 5$

D. $z = 15(x - 1) + 2(y - 2) + 5$

E. $z = 15\vec{i} + 2\vec{j}$

F. This function is not differentiable at the indicated point, so the tangent plane does not exist.

2. Consider the parametrized curve

$$\vec{r}(t) = \frac{1}{2}t^2\,\vec{i} + \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}\,\vec{j} + t\,\vec{k}, \quad 1 \leq t \leq 2.$$

What is the total arclength of this curve?

A. $\dfrac{5}{2}$

B. $\dfrac{3}{2}$

C. 0

D. 2

E. $\dfrac{19}{3}$

F. π

3. For the function $f(x,y) = xe^{xy}$, what is the value of the directional derivative of f at the point $(1,0)$ in the direction of the vector $\vec{u} = -\frac{\sqrt{2}}{2}\,\vec{i} + \frac{\sqrt{2}}{2}\,\vec{j}$?

A. -1

B. 0

C. 1

D. 2

E. 3

F. 4

4. Which of the following vector fields is conservative?

A. $\vec{F}(x,y,z) = x^2\,\vec{i} + e^y\,\vec{j} + \cos(z)\,\vec{k}$

B. $\vec{F}(x,y,z) = (x+y+z)\,\vec{i}$

C. $\vec{F}(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + xyz\,\vec{k}$

D. $\vec{F}(x,y,z) = -z\vec{i} + x\vec{k}$

E. All of the above

F. None of the above

5. Consider the function $f(x, y) = x^2 - 2y + 6x - y^2 + 2$. Which of the following statements about f is true?

A. f has a local maximum at the point $(x, y) = (-3, -1)$

B. f has a saddle point at the point $(x, y) = (-3, -1)$

C. f has a local minimum at the point $(x, y) = (-3, -1)$ and a saddle point at $(x, y) = (0, 0)$

D. f has a local maximum at the point $(x, y) = (-3, -1)$ and a saddle point at $(x, y) = (0, 0)$

E. f has a saddle point at the point $(x, y) = (-3, -1)$ and a local maximum at $(x, y) = (0, 0)$

F. f has a saddle point at the point $(x, y) = (-3, -1)$ and a local maximum at $(x, y) = (0, 0)$.

6. Which of the following expressions corresponds to the integral $\int_0^1 \int_{1-x^2}^1 f(x, y)\, dy\, dx$ with order of integration reversed?

A. $\int_1^0 \int_1^{1-x^2} f(x, y)\, dy\, dx$

B. $\int_{1-x^2}^1 \int_0^1 f(x, y)\, dx\, dy$

C. $\int_{1-x^2}^1 \int_0^1 f(y, x)\, dx\, dy$

D. $\int_0^1 \int_{1-y^2}^1 f(y, x)\, dx\, dy$

E. $\int_0^1 \int_{\sqrt{1-y}}^1 f(x, y)\, dx\, dy$

F. $\int_0^1 \int_0^1 f(x, y)\, dx\, dy$

7. Consider the vector field $\vec{F}(x,y) = (2x+2y)\,\vec{i} + (2x+2y)\,\vec{j}$, and let C be the circle $x^2+y^2=4$ oriented counter-clockwise. Then the line integral

$$\int_C \vec{F}\cdot d\vec{r}$$

is equal to

A. -3

B. -2

C. -1

D. 0

E. 1

F. 2.

8. Consider the three-dimensional solid in the **first octant** which is bounded between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$, and the cones $z=\frac{\sqrt{3}}{3}\sqrt{x^2+y^2}$ and $z=\sqrt{3}\sqrt{x^2+y^2}$. This solid has a mass-density given by $\delta(x,y,z)=x$. What is the total mass of this solid?

A. $\dfrac{5\pi}{16}$

B. $\dfrac{3\sqrt{3}-3}{4}$

C. $\dfrac{\pi}{8}$

D. 0

E. $\dfrac{\pi}{12}$

F. None of the above.

9. Consider the two-dimensional region D drawn below, whose boundary is the oriented curve C, also drawn. Let $\vec{F}(x,y) = P(x,y)\,\vec{i} + Q(x,y)\,\vec{j}$ be a vector field with continuous partial derivatives. Then which of the following equations corresponds to Green's theorem?

A. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right) dx\,dy$

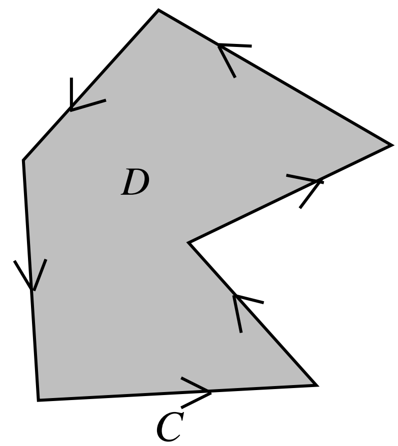
B. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}\right) dx\,dy$

C. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) dx\,dy$

D. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} + \frac{\partial P}{\partial y}\right) dx\,dy$

E. $\displaystyle\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

F. $\displaystyle\int_C Q\,dx + P\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$



10. Consider the parametrized surface S described by

$$\vec{r}(\theta, z) = 2\cos\theta\,\vec{i} + 2\sin\theta\,\vec{j} + z\,\vec{k}, \quad \frac{\pi}{4} \le \theta \le \frac{\pi}{2},\ 0 \le z \le 3,$$

and the scalar function $f(x,y,z) = yz$. What is the value of the surface integral $\displaystyle\iint_S f\,dS$?

A. $7\sqrt{2}$

B. $8\sqrt{2}$

C. $9\sqrt{2}$

D. $10\sqrt{2}$

E. $11\sqrt{2}$

F. $12\sqrt{2}$

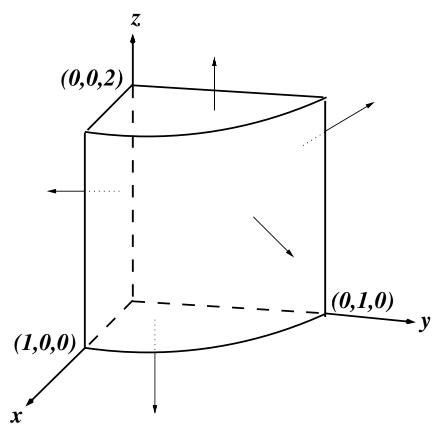
11. Consider the solid defined by the inequalities

$$0 \le z \le 8 - x^2 - y^2,$$
$$0 \le x \le 2,$$
$$0 \le y \le 1.$$

This solid has a mass density given by $\delta(x, y, z) = x + 2y$. Find the total mass of this solid.

12. Find the global extrema for the function $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the disk of radius 4, centered at the origin, $0 \leq x^2 + y^2 \leq 16$.

13. For the vector field $\vec{F}(x,y,z) = (x^2y + y\sin\, z)\,\vec{\imath} + (xy^2 + ze^x)\,\vec{\jmath} + (xy + y^2)\,\vec{k}$, compute the divergence of $\vec{F}$, i.e. compute $\vec{\nabla} \cdot \vec{F}$. Then, **using Gauss' divergence theorem**, compute the surface (flux) integral $\displaystyle\int\!\!\int_S \vec{F} \cdot d\vec{S}$, where S is the outward-oriented surface illustrated below. Note that S is the boundary of the solid region D which is the portion of the cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 2$ which lies in the first octant.



14. Consider the integral $\int_{y=0}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} e^{x^2+y^2}\, dxdy$. Convert it to polar coordinates, and then evaluate the integral.

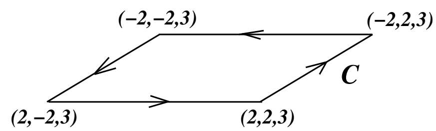
15. Consider the vector field $\vec{F}(x,y) = (2x + y\sin\, y)\,\vec{\imath} + (x\sin\, y + xy\cos\, y)\,\vec{\jmath}$.

(a) Show that the vector field $\vec{F}$ is conservative.

(b) Find a potential for $\vec{F}$, i.e. find a scalar function $f(x,y)$ such that $\vec{F}(x,y) = \vec{\nabla} f(x,y)$

(c) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the oriented straight line segment that starts at $(x,y) = (0,0)$ and ends at $(x,y) = (1, \pi/2)$.

16. Consider the vector field $\vec{F}(x,y,z) = (-yz + xe^{x^2})\,\vec{i} + (xz + y\sin(y^2))\,\vec{j} + (e^z + z\cos(z))\,\vec{k}$, and let C be the oriented curve illustrated below consisting of the four straight line segments which run from $(2,-2,3)$ to $(2,2,3)$ to $(-2,2,3)$ to $(-2,-2,3)$ and then back to $(2,-2,3)$. Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ using either a direct computation, or using Stokes' theorem. Note that one of these two methods will yield a much simpler computation than the other one, so choose the method carefully.



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